This article was downloaded by: On: 22 January 2011 Access details: Access Details: Free Access Publisher Taylor & Francis Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



To cite this Article Gent, A. N. and Hamed, G. R.(1975) 'Peel Mechanics', The Journal of Adhesion, 7: 2, 91 – 95 **To link to this Article: DOI:** 10.1080/00218467508075041 **URL:** http://dx.doi.org/10.1080/00218467508075041

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.informaworld.com/terms-and-conditions-of-access.pdf

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

J. Adhesion, 1975, Vol. 7, pp. 91–95 © 1975 Gordon and Breach Science Publishers Ltd. Printed in Scotland

Peel Mechanics

A. N. GENT and G. R. HAMED

Institute of Polymer Science, The University of Akron, Akron, Ohio 44325, U.S.A.

(Received May 31, 1974)

Published treatments of peel mechanics are shown to yield inconsistent relations for the dependence of peel force upon the angle of peel. The paradox is resolved by limiting the stress analyses to small bending deformations of the detaching strip in the still-attached region. This condition holds when the moment arm of the applied peel force is much larger than the length of the high-stress region in the bond, which must therefore be considered a prerequisite for use of the published bond stress distributions.

Failure of some experimental results to conform to the theoretical dependence of peel force upon peel angle are ascribed to inelastic deformation or stretching of the detaching strip.

1. INTRODUCTION

A sketch of a simple peel testpiece is shown in Figure 1. It consists of a flexible strip, an adhesive interlayer, and a rigid substrate. For simplicity, it is usually assumed that the flexible strip is perfectly elastic in bending, and inextensible in length, so that no energy is dissipated either in bending or extension as a portion of the strip passes through the region of detachment.

Two quite different approaches have been taken to the analysis of such a peel testpiece. The first sets up an energy balance for detachment of the adhering strip.¹⁻³ The input energy is given by

$$P_{\theta}L(1-\cos\theta)$$

where P_{θ} is the peel force, θ is the peel angle and L is the peeled length. This energy is assumed to be expended wholly in the detachment process, by an amount W_a per unit area of bonded surface. Thus,

$$W_a = P_{\theta}(1 - \cos \theta)/w,$$

where w is the width of the testpiece. The work W_a of detachment is assumed to be characteristic of the joint and independent of the peeling angle θ .



FIGURE 1 Sketch of a simple peel testpiece.

It may then be concluded that for two different peel angles θ_1 , and θ_2 the corresponding peel forces will be in the ratio:

$$P_{\theta_1}/P_{\theta_2} = (1 - \cos \theta_2)/(1 - \cos \theta_1). \tag{1}$$

For the two most common peel test angles, 90° and 180°, Eq. (1) yields

$$P_{90^{\circ}} = 2P_{180^{\circ}}.$$
 (2)

The second method of analysis of a peeling joint deals with the tensile stresses set up in the adhesive interlayer. Spies⁴ first carried out a detailed stress analysis for the special case of a peel angle of 90°; similar calculations were made later by Jouwersma⁵ and Gardon.⁶ This analysis was also generalized for other peel angles by Inoue and Kobatake⁷ and Kaelble.⁸ In all cases, the adhesive was assumed to be linearly elastic up to the point of rupture, and the theory of small bending deformations was applied to the still-attached portion of the flexible strip. The relation obtained for the peel force as a function of peel angle is:^{7,8}

$$P_{\theta} = waK_{\theta}^2 \sigma_0^2 / 2Y(1 - \cos\theta)$$
(3)

where

$$K_{\theta} = \beta m / (\beta m + \sin \theta) \tag{4}$$

and

$$\beta = (Yw/4EIa)^{\ddagger}.$$

In these equations, w is the width of the testpiece, a is the adhesive layer thickness, σ_0 is the maximum tensile stress set up in the adhesive (at the point of detachment), Y is the elastic modulus of the adhesive, EI is the bending modulus of the flexible strip, and m is the moment arm of the peel force P (Figure 1).

The quantity $\pi/2\beta$ denotes that distance into the bond at which the tensile stress decreases to zero (when $K_{\theta} = 1$). Thus, $1/\beta$ may be regarded as a dimension characteristic of the bond stress distribution, being a larger distance for less flexible strips on softer, thicker adhesive layers, and a smaller distance for more flexible strips detaching from stiffer and thinner adhesive layers.

It follows from Eqs. (3) and (4) that

$$P_{90^{\circ}} = 2K_{90^{\circ}}^2 P_{180^{\circ}}.$$

This result differs from that obtained from simple energy considerations, Eq. (2), by the factor K_{90}^2 . The two methods of analysis thus lead to quite different predictions as to the variation of peel force with peel angle for a model peel testpiece. This paradox is the subject of the present note. It is shown below that small bending theory is in fact, only applicable to peel mechanics when $K_{\theta} \simeq 1$, i.e., when $\beta m \gg \sin \theta$.

Kaelble and Ho⁹ have recently reported some experimental measurements of peel force as a function of peel angle in which they found that $P_{90^{\circ}} < 2P_{180^{\circ}}$. They attributed this result to variations of K_{θ} with peel angle, Eq. (4). In contrast, the present analysis reveals that K_{θ} is necessarily close to unity for an ideal peel testpiece, at all peel angles. Some possible reasons why Eq. (2) should fail to apply to experiments like those of Kaelble and Ho are discussed subsequently.

2. ELASTIC DEFORMATION OF A PEELING STRIP

Ι

The exact relation for the bending moment M at any section of a flexible strip is given by

$$M = EI/R = EI(d^2y/dx^2)/[1 + (dy/dx)^2]^{\frac{3}{2}},$$
 (5)

where R is the radius of curvature of the strip. In small bending theory, $(dy/dx)^2$ is regarded as negligibly small in comparison with unity, and Eq. (5) then reduces to

$$M = EI(d^2y/dx^2).$$
 (6)

Use of this simplification means that subsequent results will only be valid when $|dy/dx| \ll 1$. Although previous authors assumed that this criterion was indeed met in their analysis, the full implications of the restriction have not previously been considered.

In order to calculate an approximate value for dy/dx, we assume Eq. (6) to apply. The elongation y of the adhesive layer as a function of the distance -x into the bond is then obtained as (7, 8)

$$y = (P_{\theta}e^{\beta x}/2\beta^{3}EI)[\beta m \sin \beta x + (\beta m + \sin \theta) \cos \beta x].$$

Hence, dy/dx at x = 0 is obtained as

$$(dy/dx)_{x=0} = (P_{\theta}/2\beta^2 EI)[2\beta m + \sin \theta].$$
⁽⁷⁾

From the theory of large bending deformations of a flexible strip, the moment arm m is given by

$$m^2 = (2EI/P_{\theta})(1 - \cos \theta)$$

On substituting for P_{θ} in Eq. (7), we obtain

$$(dy/dx)_{x=0} = (1 - \cos \theta)(2\beta m + \sin \theta)/(\beta m)^2 = (1 - K_{\theta}^2)(1 - \cos \theta)/K_{\theta}^2 \sin \theta.$$
 (8)

Thus, for the limitation $|dy/dx| \leq 1$ to apply, it is apparent that K_{θ} must be close to unity at all peel angles. This means that the moment arm *m* must greatly exceed the characteristic dimension $1/\beta$ governing the bond stress distribution ($\beta m \ge 1$) so that the maximum slope of the detaching strip is sufficiently small for elementary bending theory to be a useful approximation.

To illustrate the restrictive nature of this limitation to small bending deformations, it is interesting to calculate the error involved if K is given values different from unity. For example, when K = 0.75 and $\theta = 90^{\circ}$, Eq. (8) gives a value for $(dy/dx)_{x=0}$ of 0.778. The denominator in Eq. (5) then takes the value 2.03 in place of 1. Thus, an error of about 100% is introduced into the calculated bending moment if a value of 0.75 is used for K in conjunction with small bending theory.

3. DEPENDENCE OF PEEL FORCE ON PEEL ANGLE

The original paradox is now resolved. On putting $K_{\theta} = 1$, both theoretical approaches give the same dependence of peel force on peel angle and, in particular, predict that $P_{90^{\circ}} = 2P_{180^{\circ}}$. However, this leaves unexplained some experimental observations that $P_{90^{\circ}} < 2P_{180^{\circ}}$, even when care is taken to make any kinetic factors identical in the two cases by detaching at equal rates.⁹

We conclude that the assumptions of the (less restrictive) energy theory of peeling in its simplest form were not met in these experiments. One possible reason for this may be that plastic deformation of the flexible members takes place during peeling.¹⁰ Schematic diagrams of the highly-stressed portions of strips peeled off at 90° and 180° are shown in Figure 2. It seems reasonable to assume that the amount of plastic strain undergone by the strip for a 180° peel is about twice that for a 90° peel. Thus, about twice as much work would be expended in plastic deformation at 180° and the total work required to peel would be correspondingly greater at 180° than at 90°.

This hypothesis is also in accord with the observation⁹ that at lower rates



FIGURE 2 Plastic strains in peeling at 90° and 180° (schematic).

of peeling the behavior approaches more nearly that expected from theory, i.e., $P_{90^\circ} = 2P_{180^\circ}$, because at lower rates the peel forces are smaller and plastic yielding, if present, is less important.¹¹

An alternative reason (suggested by one of the reviewers) why Eq. (1) may not hold is that the detached strip may stretch elastically after detachment and work will then be expended in stretching it in addition to the work of detachment. The work of stretching will depend upon the magnitude of the peel force and the elastic properties of the strip. It will be different for different peel angles, in a predictable way but not in accordance with Eq. (1). The dependence has been discussed recently by Lindley.¹²

In any event, deviations from Eq. (1) must be ascribed to inelastic behavior of the peel testpiece or to subsequent deformation of the adherends, and not to stress distributions set up in the process of peeling apart elastic components.

Acknowledgement

This work forms part of a program of research supported by a grant from the Engineering Division of the National Science Foundation.

References

- 1. R. S. Rivlin, Paint Technol. 9, 1 (1944).
- B. V. Deryagin and N. A. Krotova, Doklady Akad. Nauk SSSR 61, 849 (1948); Chem. Abs. 43, 2842 (1949).
- 3. J. J. Bikerman, *The Science of Adhesive Joints*, 2nd ed. (Academic Press, New York, 1968).
- 4. G. J. Spies, Aircraft Engng. 25, 64 (1953).
- 5. C. Jouwersma, J. Polym. Sci. 45, 253 (1960).
- 6. J. L. Gardon, J. Appl. Polym. Sci. 7, 643 (1963).
- 7. Y. Inoue and Y. Kobatake, Appl. Sci. Res. A8, 321 (1959).
- 8. D. H. Kaelble, Trans. Soc. Rheol. 4, 45 (1960).
- 9. D. H. Kaelble and C. L. Ho, Trans. Soc. Rheol. 18, 219 (1974).
- 10. A. J. Duke and R. P. Stanbridge, J. Appl. Polym. Sci. 12, 1487 (1968).
- A. N. Gent and J. Schultz, Proc. Internatl. Rubber Conf., Brighton, England, May 1972 (Institution Rubber Industry, London, 1973). Pp. Cl. 1-6.
- 12. P. B. Lindley, J. Instn. Rubber Industr. 5, 243 (1971).